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ENGINEERING SEISMIC RISK ANALYSIS

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ABSTRACT

This paper introduces a method for the evaluation of the seismic risk at the site of an engineering project. The results are in terms of a ground motion parameter (such as peak acceleration) versus average return period. The method incorporates the influence of all potential sources of earthquakes and the average activity rates assigned to them. Arbitrary geographical relationships between the site and potential point, line, or areal sources can be modeled with computational ease. In the range of interest, the derived distributions of maximum annual ground motions are in the form of Type I or Type II extreme value distributions, if the more commonly assumed magnitude distribution and attenuation laws are used.

INTRODUCTION

Owing to the uncertainty in the number, sizes, and locations of future earthquakes it is appropriate that engineers express seismic risk, as design winds or floods are, in terms of return periods (Blume, 1965; Newmark, 1967; Blume, Newmark and Corning, 1961; Housner, 1952; Muto, Bailey and Mitchell, 1963; Gzovsky, 1962).

The engineer professionally responsible for the aseismic design of a project must make a fundamental trade-off between costly higher resistances and higher risks of economic loss (Blume, 1965). It requires assessment of the various levels of performance and economic implications of particular designs subjected to various levels of intensity of ground motion. The engineer must consider the performance of the system under moderate as well as large motions. Sound design often suggests some economic loss (e.g., architectural damage in buildings, automatic shut-down costs in nuclear power plants) under these moderate, not unexpected earthquake effects.

This engineer should have available all the pertinent data and professional judgement of those trained in seismology and geology in a form most suitable for making this decision wisely. This information is far more usefully and completely transmitted through a plot of, say, Modified Mercalli intensity versus average return period than through such ill-defined single numbers as the "probable maximum" or the "maximum credible" intensity. Even well-defined single numbers such as the "expected lifetime maximum" or "50-year" intensity are insufficient to give the engineer an understanding of how quickly the risk decreases as the ground motion intensity increases. Such information is crucial to well-balanced engineering designs, whether it is used informally and intuitively (Newmark, 1967), more systematically (Blume, 1965), or directly in statistically-based optimization studies (Sandi, 1966; Benjamin, 1967; Borgman, 1963).

Unfortunately it has not been a simple matter for the seismologist to assess and express the risk at a site in these terms. He must synthesize historical data, geological information, and other factors in this assessment. The locations and activities of potential sources of tectonic earthquakes may be many and different in kind; they may not even be well known. In some regions, for example, it is not possible to correlate past activity with known geological structure. In such circumstances the seismologist understandably has been led to express his professional opinion in terms of one or two single numbers, seldom quantitatively defined. It is undoubtedly difficult, in this situation, for the seismologist to avoid engineering influences; the seismologist's estimates will probably be more conservative for more consequential projects. But these decisions are more appropriately those of the design engineer who has at hand more complete information (such as construction costs, system performance characteristics, etc.) upon which to determine the optimal balance of cost, performance, and risk.

Seismologists have long recognized this need to provide engineers with their best estimates of the seismic risk. Numerous regional seismic zoning maps have been developed. Familiar examples appear in the Uniform Building Code (1967) and Richter (1959). Despite reference to probabilities they are seldom clear as to how the (single) intensity level for each location is to be interpreted. More recently these values have been associated with specific average return periods (Muto, Bailey and Mitchell, 1963; Kawasumi, 1951; Ipek *et al*, 1965). In any case, more information is needed to define a relationship between a continuous range of average return period and intensities. Other attempts have been made to provide this more complete information at regional levels (Ipek, 1965; Milne and Davenport, 1965). These approaches, which are usually large scale numerical studies based directly on historical data, have difficulty giving proper weight to the known correlation between geological structure and most seismic activity. They also are not successful at a fine or local scale. Lacer (1965) has presented a numerical, Monte Carlo technique designed to estimate the distribution of the intensity of motion at a particular site given the occurrence of an earthquake somewhere in the surrounding region. He is able to account for geological features, such as faults, but he assumes all the assigned "point" sources are equal likely to give rise to this earthquake.

In this paper a method is developed to produce for the engineer the desired relationships between such ground-motion parameters as Modified Mercalli Intensity, peak-ground velocity, peak-ground acceleration, etc., and their average return period for his site. The minimum data needed are only the seismologist's best estimates of the average activity levels of the various potential sources of earthquakes (e.g., a particular fault's average annual number of earthquakes in excess of some minimum magnitude of interest, say 4). If, in addition, the seismologist has reason to use other than average or typical values of the parameters in the function used to describe the relative frequency of earthquake magnitudes or in the functions of intensity, say, versus magnitude and distance, he may also supply these parameter values. The technique to be developed provides the method for integrating the individual influences of potential earthquake sources, near and far, more active or less, into the probability distribution of maximum annual intensity (or peak-ground acceleration, etc.). The average return period follows directly. The results of the development appear in closed analytical form, requiring no lengthy computation and permitting direct observation of the sensitivity of the final results to the estimates made.

Unlike the analogous flood or wind problem, in the determination of the distribution of the maximum annual earthquake intensity at a site, one must consider not only the distribution of the size (magnitude) of an event, but also its uncertain distance from the site and the uncertain number of events in any time period. The presentation here will show the mathematical development of a simple case. Results of other cases of interest will be displayed without complete derivations. An illustration will demonstrate the application of the method. Finally, the assumptions and limitations will be discussed more critically. Extensions and advantages of the method will conclude the presentation.

Line Source Derivation

For illustration of the development of the method of solution, the determination of the distribution of the annual maximum Modified Mercalli intensity at a site due to potential earthquakes along a neighboring fault will be considered. As illustrated in

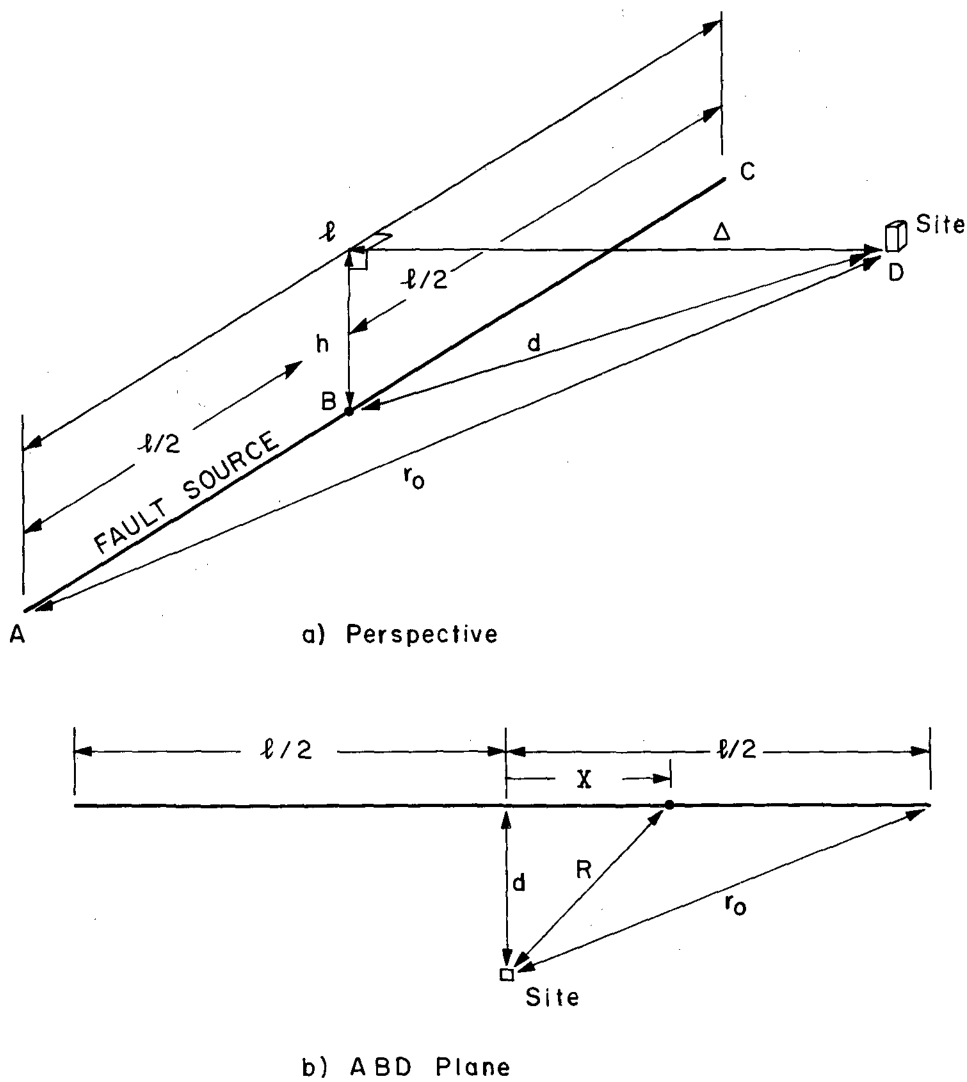


FIG. 1. Line source.

Figure 1a, the site is assumed to lie a perpendicular distance, Δ, from a line on the surface vertically above the fault at the focal depth, h, along which future earthquake foci are expected to lie. The length of this fault is l, and the site is located symmetrically with respect to this length.

Concern with focal distances restricts attention to the ABD plane, Figure 1b. The perpendicular slant distance to the source is

$$d = \sqrt{h^2 + \Delta^2} \tag{1}$$

The focal distance, R, to any future focus located a distance X from the point B is

$$R = \sqrt{d^2 + X^2} \tag{2}$$

Since $-l/2 \leqq X \leqq l/2$, the distance to any earthquake focus is restricted to $0 \leqq R < r_0$ in which $r_0 = \sqrt{d^2 + l^2/4}$. In general the size and location of a future earthquake are uncertain. They shall be treated therefore as random variables. (Random variables are denoted by capital letters.)

We first seek the conditional distribution of the Modified Mercalli Intensity, I, at the site given that an earthquake occurs at a focal distance $R = r$ from the site. For illustration we us the common assumption (Ipek, 1965; Esteva and Rosenblueth, 1964; Wiggins, 1964; Kanai, 1961) that in the range of interest the intensity has the following dependence on magnitude, M, and focal distance, R:

$$I = c_1 + c_2 M - c_3 \ln R \tag{3}$$

in which ln denotes natural logarithm and c_i, $i = 1, 2, 3$, are semiempirical constants on the order of 8, 1.5, and 2.5, respectively for firm ground in southern California (Esteva and Rosenblueth, 1964).

Given that an earthquake occurs at focal distance $R = r$, the probability that I, the intensity at the site, is greater than any number i is, using equation 3,

$$P[I \geqq i \mid R = r] = P[c_1 + c_2 M - c_3 \ln r \geqq i \mid R = r] \tag{4}$$

in which $P[A \mid B]$ is read the probability of A given B. Assuming probabilistic independence of M and R,

$$\begin{aligned} P[I \geqq i \mid R = r] &= P\left[M \geqq \frac{i + c_3 \ln r + c_1}{c_2}\right] \\ &= 1 - F_M\left[\frac{i + c_3 \ln r + c_1}{c_2}\right] \end{aligned} \tag{5}$$

in which $F_M(m)$ is the cumulative distribution function of earthquake magnitudes. For example, Richter's widely verified (19, 20) relationship between number, n_m, and magnitude, m

$$\log_{10} n_m = a - bm$$

implies

$$1 - F_M(m) = e^{-\beta(m-m_0)} \qquad m \geqq m_0 \tag{6}$$

in which $\beta = b \ln 10$ and m_0 is some magnitude small enough, say 4, that events of lesser magnitude may be ignored by engineers. This restriction to larger events implies that the probabilities above are conditional on the occurrence of an event of interest, that is, one where $M \geqq m_0$. The parameter b is typically (Isacks and Oliver, 1964) such that β is about 1.5 to 2.3.

Combining equations 5 and 6, the result is

$$P[I \geqq i \mid R = r] = \exp\left[-\beta\left[\frac{i + c_3 \ln r + c_1}{c_2} - m_0\right]\right]. \qquad (7)$$

The limit on the definition of $F_M(m)$, namely $m \geqq m_0$, implies that equation 7 holds for

$$\frac{i + c_3 \ln r + c_1}{c_2} \geqq m_0$$

or

$$i \geqq c_2 m_0 - c_1 - c_3 \ln r. \qquad (8)$$

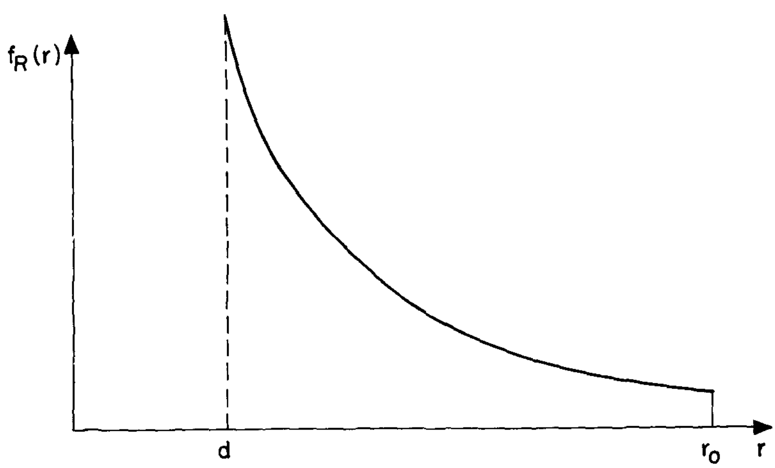


FIG. 2. Probability density function of focal distance, R.

At smaller values of the argument, i, the probability (equation 7) is unity that I exceeds i (given the occurrence of an event of magnitude greater than m_0 at distance r).

In order to consider the influence of all possible values of the focal distance and their relative likelihoods, we must integrate. We seek the cumulative distribution of I, $F_I(i)$, given an occurrence of $M \geqq m_0$,

$$1 - F_I(i) = P[I \geqq i] = \int_d^{r_0} P[I \geqq i \mid R = r] f_R(r)\, dr \qquad (9)$$

in which $f_R(r)$ is the probability density function of R, the uncertain focal distance.

For the illustration here, it is assumed that, given an occurrence of an event of interest along the fault, it is equally likely to occur anywhere along the fault. Formally, the location variable X is assumed to be uniformly distributed on the interval $(-l/2, +l/2)$. Thus $\mid X \mid$, the absolute magnitude of X, is uniformly distributed on the interval $(0, l/2)$. The cumulative probability distribution, $F_R(r)$, of R follows immediately:

$$F_R(r) = P[R \leqq r] = P[R^2 \leqq r^2]$$
$$= P[X^2 + d^2 \leqq r^2] = P[\,|X| \leqq \sqrt{r^2 - d^2}]$$
$$= \frac{r^2 - d^2}{l/2} \qquad d \leqq r \leqq r_0 . \tag{10}$$

Therefore, the probability density function of R is

$$f_R(r) = \frac{dF_R(r)}{dr} = \frac{d}{dr}\left(\frac{2\sqrt{r^2 - d^2}}{l}\right)$$
$$= \frac{2r}{l\sqrt{r^2 - d^2}} \qquad d \leqq r \leqq r_0 . \tag{11}$$

This density function is plotted in Figure 2.

Substituting equation 11 into equation 9 and integrating is complicated by the awkward limits of definition of the functions, but in the region of greatest interest, namely larger values of the intensity the result is

$$1 - F_I(i) = P[\ \geqq i]$$
$$= \frac{1}{l}\, CG \exp\left[-\frac{\beta}{c_2}\, i\right] \qquad i \geqq i' \tag{12}$$

in which i' is the lower limit of validity of this form of the result and equals

$$i' = c_1 + c_2 m_0 - c_3 \ln d \tag{13}$$

and in which C and G are constants. The first constant is related to parameters in the various relationships used above:

$$C = \exp\left[\beta\left(\frac{c_1}{c_2} + m_0\right)\right]. \tag{14}$$

The second constant is related to the geometry of illustration:

$$G = 2\int_d^{r_0} \frac{dr}{r^\gamma\sqrt{r^2 - d^2}}$$
$$= \frac{2}{d^\gamma}\int_0^{\sec^{-1}[r_0/d]} (\cos u)^{\gamma-1}\, du \tag{15}$$

in which

$$\gamma = \beta\,\frac{c_3}{c_2} - 1. \tag{16}$$

The integral in equation 15 must be evaluated numerically. Results appear in Figure 3. For typical parameter values and sufficiently long faults it is conservative and reasonable to replace r_0 by infinity. In this case G is given by

$$G = \frac{2\pi}{(2d)^\gamma} \frac{\Gamma(\gamma)}{\left[\Gamma\left(\frac{\gamma+1}{2}\right)\right]^2} \tag{17}$$

in which $\Gamma(\gamma)$ is the complete gamma function and γ is restricted to positive values.

The results above yield the probability that the site intensity, I, will exceed a

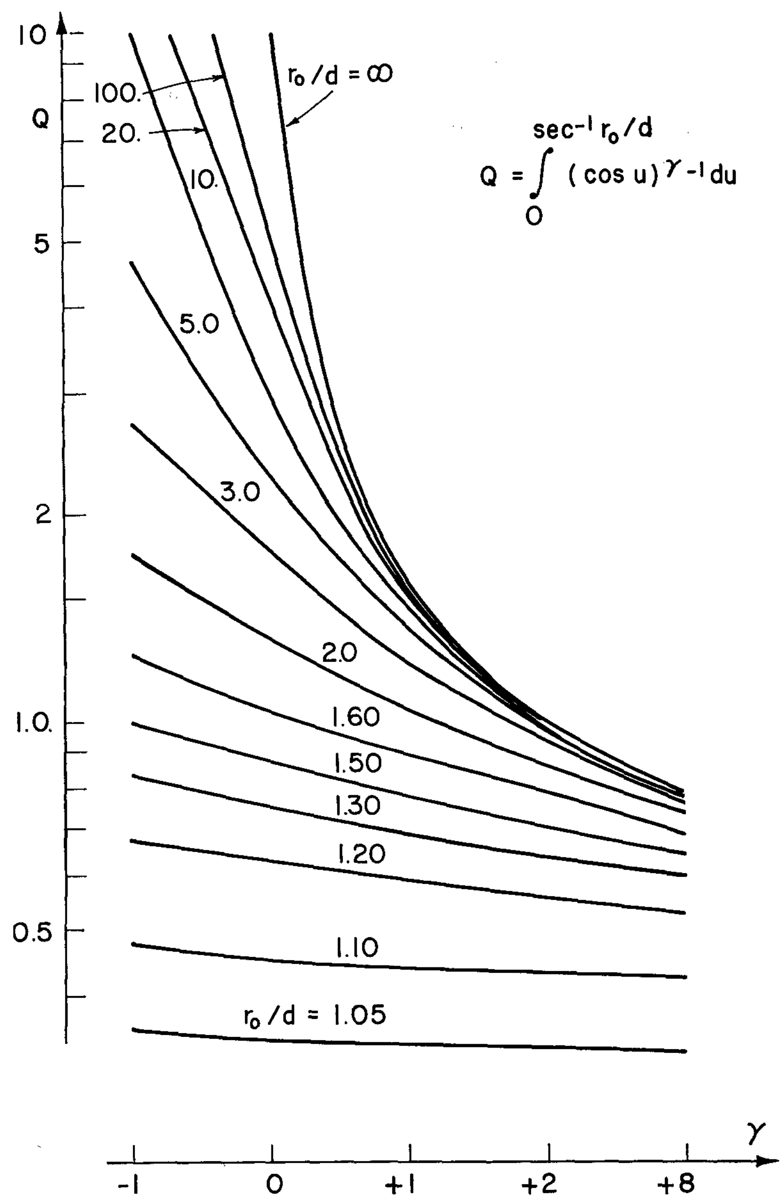


FIG. 3. Numerical values of integral in equation (15).

certain value, i, given that an event of interest ($M \geqq m_0$) occurs somewhere along the fault. Next we must consider the question of the random number of occurrences in any time period. For illustration, it is assumed that the occurrences of these major events follow a Poisson arrival process (Parzen, 1962; Cornell, 1964) with average occurrence rate (along the entire fault) of ν per year. Then, $\tilde{N}$, the number of events of interest along the fault in a time interval of length t years is known to be Poisson distributed

$$p_{\tilde{N}}(n) = P[\tilde{N} = n] = \frac{e^{-\nu t}(\nu t)^n}{n!} \qquad n = 0, 1, 2, \cdots . \tag{18}$$

It is easily established that, if certain events are Poisson arrivals with average arrival rate ν and if each of these events is independently, with probability p, a "special event," then these special events are Poisson arrivals with average rate $p\nu$. (This is said to be a Poisson process with ("random selection.") In our case the special events are those which cause an intensity at the site in excess of some value i. The probability, p_i, that any event of interest ($M \geqq m_0$) will be a special event is given by equation 12.

$$p_i = P[I \geqq i] = \frac{1}{l} CG \exp\left[\frac{-\beta}{c_2} i\right]. \tag{19}$$

Thus the number of times N that the intensity at the site will exceed i in an interval of length t is

$$p_N(n) = P[N = n] = \frac{e^{-p_i \nu t}(p_i \nu t)^n}{n!} \qquad n = 0, 1, 2, \cdots . \tag{20}$$

Such probabilities are useful in studying losses due to a succession of moderate intensities or cumulative damage due to two or more major ground motions.

Of particular interest is the probability distribution of $I_{\max}^{(t)}$ the maximum intensity over an interval of time t (often one year). Observe that

$P[I_{\max}^{(t)} \leqq i] = P$[exactly zero special events in excess of i occur in the time interval 0 to t]

which from equation (20) is

$$P[I_{\max}^{(t)} \leqq i] = P[N = 0] = e^{-p_i \nu t}. \tag{21}$$

If we let $I_{\max}$ equal $I_{\max}^{(1)}$, the *annual* maximum intensity, $t = 1$, and

$$F_{I_{\max}^{(i)}} = e^{-p_i \nu} = \exp\left[-\hat{\nu} CG \exp\left(-\frac{\beta}{c_2} i\right)\right] \qquad i \geqq i' \tag{22}$$

in which now the ratio $\hat{\nu} = \nu/l$ appears. This ratio is the average number of occurrences per unit length per year.

The conclusion is that for the larger intensities of engineering interest, the annual maximum intensity has a distribution of the double exponential or Gumbel type. This distribution is widely used in engineering studies of extreme events. It is important to realize that, here, this conclusion is *not* based on the intuitive appeal to the familiar asymptotic extreme value argument (Gumbel, 1958), which has caused other investigators to seek and find empirical verification of the distribution for maximum magnitudes or intensities in a given region (Milne and Davenport, 1965; Nordquist, 1945; Dick, 1965). The form of the distribution is dependent on the functional form of the various relationships assumed above. Others, too, have found (Dick, 1965; Epstein and Lomnitz, 1966; Epstein and Brooks, 1948) that the combination of Poisson occurrences of events and exponentially distributed "sizes" of events will invariably lead to the conclusion that the largest event has a Gumbel-like distribution (the true Gumbel distribution is non-zero for negative as well as positive values of the argument). Any combination of assumptions which leads to the exponential form of the distribution of I will, in combination with Poisson assumption of event occurrences, yield this Gumbel distribution. The exponential form of $F_I(i)$ does not require the exponential form of $F_M(m)$. If the logarithmic dependence of I on R (equation 3) is retained, for example, even polynomial distributions (Housner, 1952) of magnitude will lead to the exponential distribution of I.

If the annual probabilities of exceedance are small enough (say $\leqq 0.05$), the distribution of $I_{\max}$ can be approximated by

$$\begin{aligned} 1 - F_{I_{\max}}(i) &= 1 - e^{-p_i \nu} \cong 1 - (1 - p_i \nu) \\ &\cong p_i \nu \\ &\cong \nu C G \exp\left(-\frac{\beta}{c_2} i\right) \qquad i \geqq i'. \end{aligned} \tag{23}$$

The average return period, T_i, of an intensity equal to or greater than i is defined as the reciprocal of $1 - F_{I_{\max}}(i)$ or

$$T_i \cong \frac{1}{\nu C G} \exp\left(\frac{\beta}{c_2} i\right) \qquad i \geqq i' \tag{24}$$

or, the "T-year" intensity is

$$i \cong \frac{c_2}{\beta} \ln (\nu C G T_i) \qquad i \geqq i'. \tag{25}$$

Consider the following typical numerical values of the parameters and site constants, applicable to a particular site in Turkey, where in one region in 1953 years it was found (Ipek *et al*, 1965) that

$$\begin{aligned} \log_{10} n_m &= a - bm \\ &= 5.51 - 0.644m \end{aligned}$$

in which n_m is the number of earthquakes greater than m in magnitude. Assuming these earthquakes all occur along the 650 km of the major fault system in the region, the average number of earthquakes in excess of magnitude 5 (i.e., $m_0 = 5$) per year per unit length of fault is

$$\hat{\nu} = \frac{n_5}{(1953)(650)} = 1.5 \times 10^{-4} \text{ (year)}^{-1} \text{ (kilometer)}^{-1}.$$

Also

$$\beta = b \ln 10 = 0.644(2.30) = 1.48.$$

Using attenuation constants found empirically (Esteva and Rosenblueth, 1964) for California

$$c_1 = 8.16$$

$$c_2 = 1.45$$

$$c_3 = 2.46$$

the following numerical results are obtained for a site located a minimum surface distance, Δ, of 40 km from a line source of earthquakes at depth $h = 20$ km:

$$d = \sqrt{h^2 + \Delta^2} = 44.6 \text{ km}$$

$$\gamma = \beta \frac{c_3}{c_2} - 1 = 1.52$$

$$C = \exp\left[\beta\left(\frac{c_1}{c_2} + m_0\right)\right] = 6.85 \times 10^6$$

$$G \cong \frac{2\pi}{(2d)^\gamma} \frac{\Gamma(\gamma)}{\left[\Gamma\left(\frac{\gamma+1}{2}\right)\right]^2} = 7.04 \times 10^{-3}.$$

(Numerical integration gives $G = 6.58 \times 10^{-3}$). Thus, the intensity at this site with return period T_i is

$$i \cong \frac{c_2}{\beta} \ln (\hat{\nu} C G T_i)$$

$$\cong 0.98 \ln (6.9 T_i).$$

Note the logarithmic relationship between i and T_i. The risk that a design intensity will be exceeded can be halved (T doubled) by increasing the design intensity by about 0.7. This equation is plotted in Figure 4 for the range of validity $i \geqq i'$ where

$$i' = c_1 + c_2 m_0 - c_3 \ln d = 6.08.$$

If interest extends to smaller intensities, it necessitates more cumbersome integrations not shown here.

Peak Ground Motion Results

The previous section developed the desired distribution results for the Modified Mercalli intensity, I, and a uniform line source, with a particular set of assumptions on magnitude distribution and the intensity versus M and R relationship. Engineers are generally more directly concerned with such ground motion parameters as peak-ground acceleration, A, peak-ground velocity, V, or peak-ground displacement, D, than with intensity itself.

An argument parallel to that in the preceding section can be carried out with any functional relationship between the site ground-motion variable, Y, and M and R. For example, the particular form

$$Y = b_1 e^{b_2 M} R^{-b_3} \tag{26}$$

has been recommended by Kanai (1961) and by Esteva and Rosenblueth (1964)* for peak-ground acceleration ($Y = A$), peak-ground velocity ($Y = V$), and peak-ground displacement ($Y = D$). The latter authors (Esteva and Rosenblueth, 1964; Esteva,

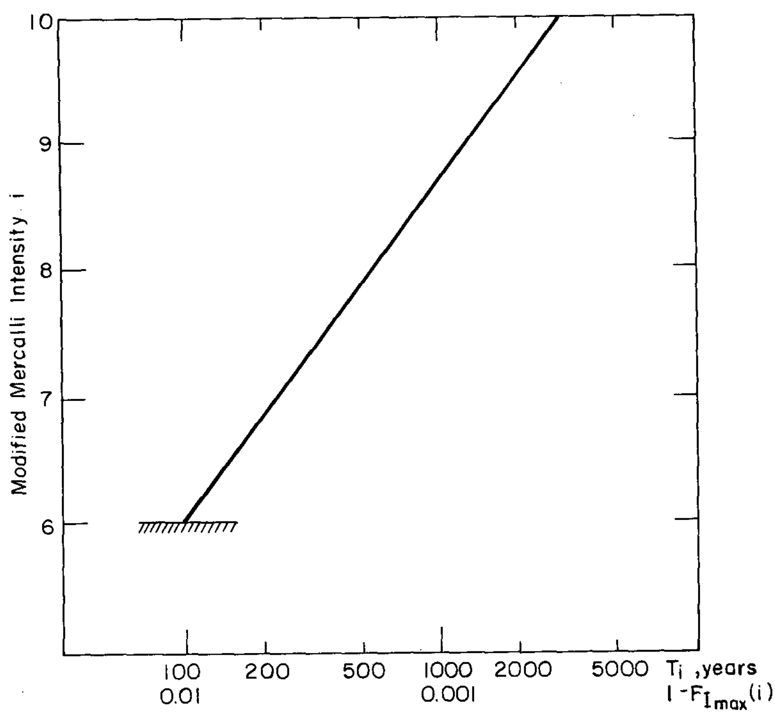


FIG. 4. Numerical example: Intensity versus return period.

1967) (on theoretical and empirical grounds) suggest that the constants $\{b_1, b_2, b_3\}$ be $\{2000, 0.8, 2\}$, $\{16, 1.0, 1.7\}$, and $\{7, 1.2, 1.6\}$ for A, V, and D respectively in southern California, with A, V, and D in units of centimeters and seconds and R in kilometers.

For the general relationship in equation 26, an argument like that in the previous section yields for the annual maximum value of Y from a uniform line source

$$F_{Y_{\max}}(y) = \exp\left[-\nu CG y^{-\beta/b_2}\right] \qquad y \geqq y' \tag{27}$$

$$1 - F_{Y_{\max}}(y) \cong \nu CG y^{-\beta/b_2} \qquad y \geqq y' \tag{28}$$

$$T_y \cong \frac{1}{\nu CG} y^{\beta/b_2} \tag{29}$$

*More recently, Esteva (1967), it has been suggested that the focal depth, h, in kilometers, be replaced by an empirically adjusted value, $\sqrt{h^2 + 20^2}$, which increases the formula's accuracy shorter focal distances.

in which

$$C = e^{\beta m_0} b_1^{\beta/b_2} \tag{30}$$

and G is as given in equation 15 (or equation 17) with

$$\gamma = \beta \frac{b_3}{b_2} - 1\,, \tag{31}$$

The lower limit of the validity of these forms of $F_{Y_{\max}}(y)$ is

$$y' = b_1 e^{b_2 m_0} d^{-b_3}. \tag{32}$$

For durations, t, other than one year, $\bar{\nu}$ should be replaced by $\bar{\nu}t$ in equations 27 and 28.

Notice that equation 27 is of the general form of the Type II asymptotic extreme value distribution of largest values (Gumbel, 1958). This distribution, too, is commonly used in the description of natural loadings on engineering structures, the most familiar being maximum annual wind velocities (Task Commission on Wind Forces, 1961; Thom, 1967). The justification there is based on asymptotic (large N) arguments while that here is not. The results here are a consequence of the forms of the relationships assumed.

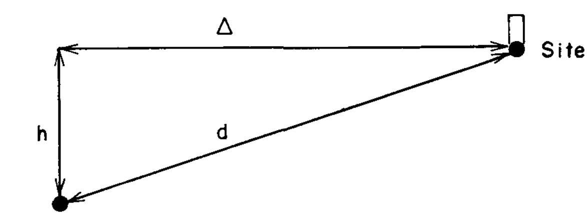


FIG. 5. Point source, cross section.

Using results such as these the designer can compute for his site the peak-ground velocity, v, and peak-ground acceleration, a, associated with the same, say the 200-year, return period. For the numerical example in the previous section and the values of the parameters referred to in this section, these values are approximately

$$v = 7.5 \text{ cm/sec} = 3 \text{ in/sec}$$

$$a = 80 \text{ cm/sec}^2 = 0.08\text{g}.$$

GENERAL SOURCE RESULTS

In order to facilitate representing the geometry and potential source conditions at arbitrary sites, it is desirable to have additional results for point and area sources. It will be shown that these results can be used to represent quite general conditions.

If a potential source of earthquakes is closely concentrated in space relative to its distance, d, from the site, it satisfactorily may be assumed to be a point source, Figure 5. (Examples might be sites one or two hundred kilometers from New Madrid, Mo. or Charleston, S. C.) In this case there is no uncertainty in the focal distance, d, and the previous results (e.g., equations 22, 25, 27, 29) hold with $\hat{\nu}$ equal to the average number of earthquakes of interest ($M \geqq m_0$) per year originating at this point and with a geometry term (in place of equation 15) equal to

$$G = d^{-(\gamma+1)} \tag{33}$$

For intensities γ is given by equation 16 and for variables with relationships of the type shown in equation 26, γ is given by equation 31. For a point source, for values of the argument less than i' or y', the cumulative distribution function (equation 22 or 27) is simply zero.

In some situations, owing to an apparent lack of correlation between geologic structure and seismic activity or owing to an inability to observe this structure due to deep overburdens, it may be necessary for engineering purposes to treat an area surrounding the site as if earthquakes were equally likely to occur anywhere over the area. It can

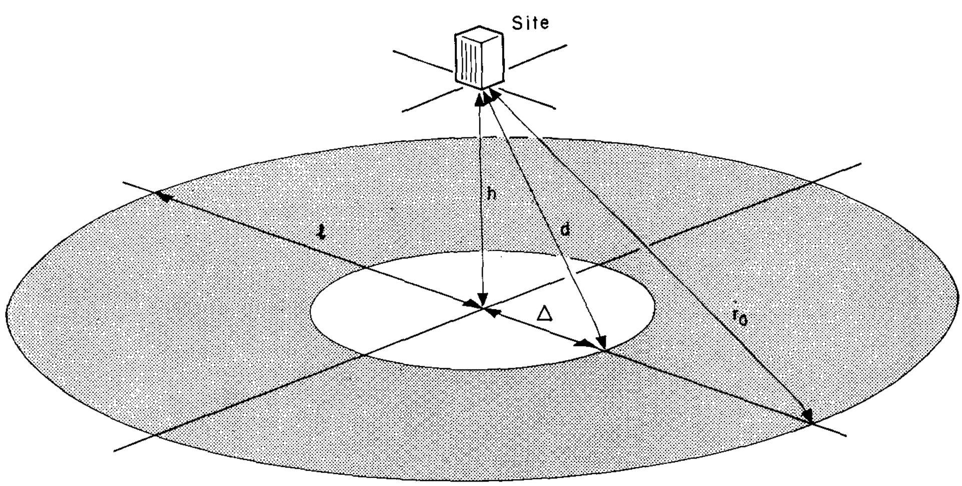


FIG. 6. Annular sources, perspective.

be shown that for an annular areal source surrounding the site, as pictured in Figure 6, the distributions above (equations 22 and 27) hold with a geometry term equal to

$$G = \frac{2\pi}{(\gamma - 1)d^{\gamma-1}} \left[1 - \left(\frac{r_0}{d} \right)^{-(\gamma-1)} \right] \tag{34}$$

with γ given by equation 16 or 31. The value of $\hat{\nu}$ should now be the average number of earthquakes of interest ($M \geqq m_0$) per year per unit area. In terms of ν, the average number per year over the entire annular region, $\hat{\nu}$ is

$$\hat{\nu} = \frac{\nu}{\pi(l^2 - \Delta^2)} . \tag{35}$$

For values of the argument less than i' or y', the cumulative distribution function (equation 22 or 27) is zero. Note that d will never be less than h. Thus the geometry factor remains finite even when the site is "immersed" in the areal source, i.e., when $\Delta = 0$, and an earthquake directly below the site is an (improbable) possibility.

When more complex source configurations exist, the distribution function for the maximum value of some ground motion variable can be found by combining the results above. For example, if there exist independent sources $(1, 2, \cdots n)$ of the various types discussed above, the probability that the maximum value of Y, the peak-ground acceleration, for example, is less than y is the probability that the maximum values from sources 1 through n are *all* less than y, or

$$F_{Y_{\max}^{(y)}} = F_{Y_{\max_1}^{(y)}} F_{Y_{\max_2}^{(y)}} \cdots F_{Y_{\max_n}^{(y)}}$$

$$= \prod_{j=1}^{n} F_{Y_{\max_j}^{(y)}}$$

in which $F_{Y_{\max_j}^{(y)}}$ is the distribution of the maximum Y (say peak acceleration) from source j, as given by equation 27 with the appropriate values of the parameters ν_j, C_j, G_j. Note that the different possible focal depths on the same fault can be accounted for in this manner.

For the exponential form of the $F_{Y_i}^{(y)}$ functions (equation 27)

$$F_{Y_{\max}^{(y)}} = \exp\left[-\sum_{j=1}^{n} \nu_j C_j G_j y^{-\beta_j/b_{2_j}}\right] \qquad y > y' \qquad (36)$$

where y' is the largest of the y_j'. For y less than y', the distribution can be found with ease (unless a line source is involved). If the constants β, b_1, b_2, b_3 are the same for all the sources in the region around the site, equation 36 becomes simply

$$F_{Y_{\max}^{(y)}} = \exp\left[-C\nu G y^{-\beta/b_2}\right] \qquad y > y' \qquad (37)$$

in which

$$\nu G = \sum_{j=1}^{n} \nu_j G_j \qquad (38)$$

A similar conclusion holds for Modified Mercalli intensities, equation 22.

In short the distributions retain the same forms with the product, νG, equal to the sum of the corresponding products over the various sources. With respect to these products, then, linear superposition applies. This conclusion is a reflection of the fact that the sum of independent Poisson process is a Poisson process with an average arrival rate equal to the sum of individual rates.

This conclusion can be used to determine geometry factors for unsymmetrical source geometries. For example, for the condition in Figure 7a, the geometry factor, G, must equal one-half of that for the symmetrical situation. The geometry factor for the situation in Figure 7b must equal one-half of that for a symmetrical source length $2b$ minus one-half of that for a symmetrical source of length $2a$, or

$$G = \tfrac{1}{2}[G' - G''] \qquad (39)$$

in which G' and G'' are calculated from equation 15 with values r_0' and r_0'' respectively. An example will follow. This result also permits easy treatment of a fault with a (spatially) non-constant average occurrence rate, each different portion of the fault being treated independently.

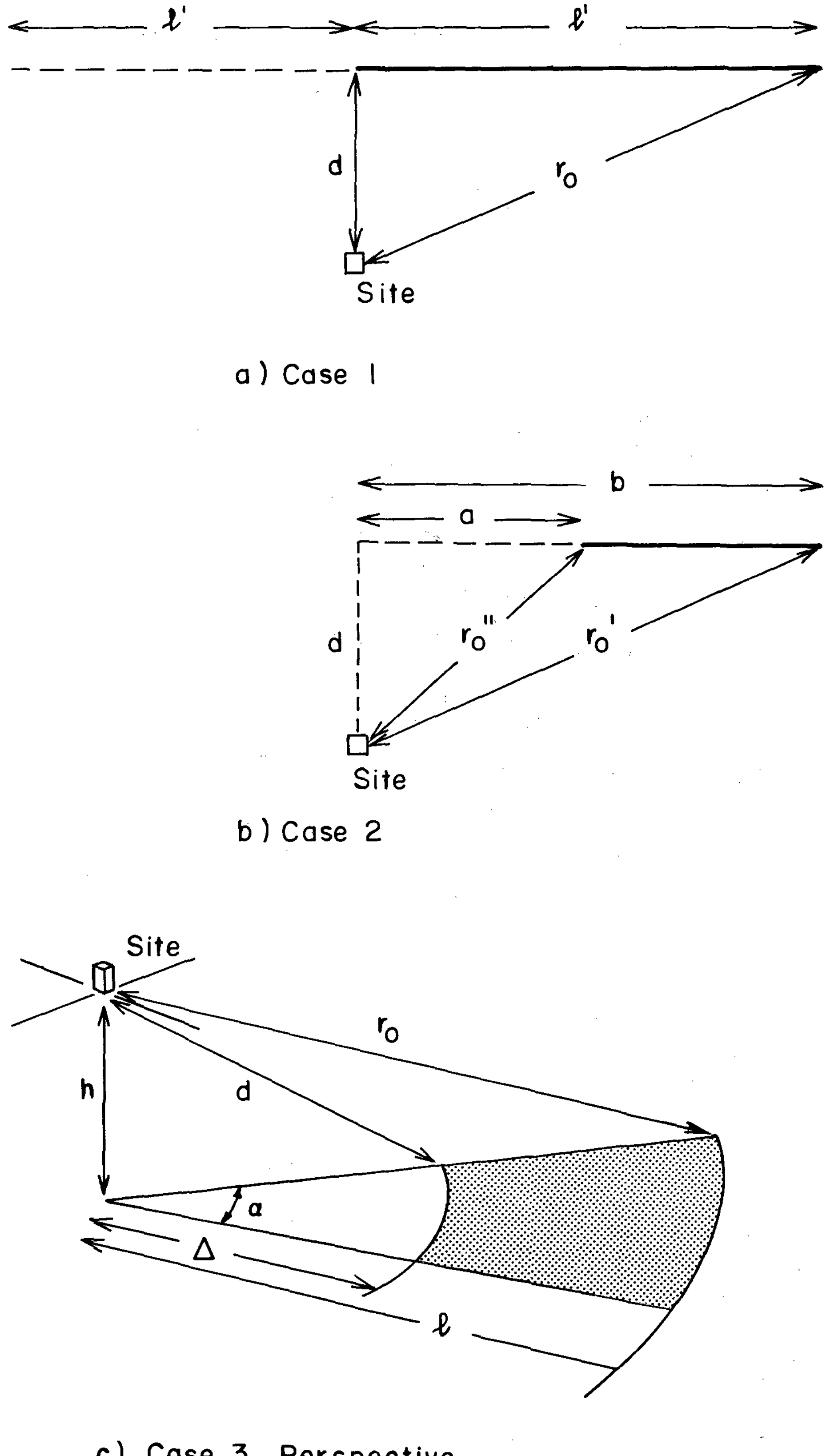


Fig. 7. Unsymmetrical sources.

In the same manner the geometry factor for an area such as that shown in Figure 7c is found to be

$$G_\alpha = \frac{\alpha}{2\pi} G_{2\pi}$$

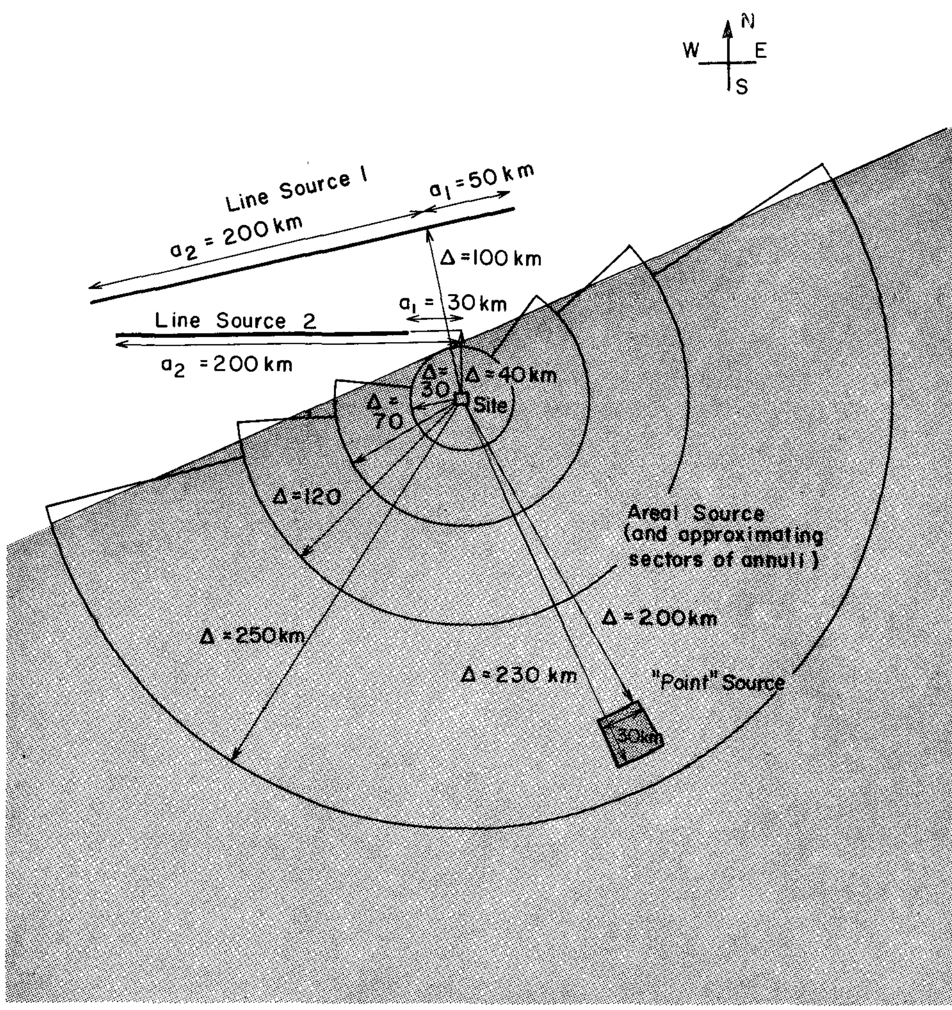


FIG. 8. Numerical examples: plan

in which $G_{2\pi}$ is the result for the complete annulus, equation 34. As will be shown in a numerical example, an areal source of arbitrary shape can be modeled with ease by approximating it by a number of such shapes.

Note that the approximation to equation 37 for smaller values of the probability

$1 - F_{Y_{\max}(y)}$ becomes

$$1 - F_{Y_{\max}(y)} \cong C y^{-\beta/b_2} \sum_{j=1}^{n} \nu_j G_j \tag{41}$$

suggesting that the (small) probability that the annual maximum, $Y_{\max}$, exceeds y in any year is made up of the sum of the probabilities contributed by each of the sources. Also, for larger values the return period is approximately

$$T_y \cong \frac{1}{C\Sigma\nu_j G_j}\, y^{\beta/b_2}. \tag{42}$$

TABLE 1, PART 1

NUMERICAL EXAMPLE

Source	d	r_0	G_i		
			A	V	D
Line 1					
Right portion	104	115.3	5.12×10^{-7}	1.73×10^{-4}	3.66×10^{-3}
Left portion	104	241	1.06×10^{-6}	3.17×10^{-4}	5.99×10^{-3}
Line 2					
Total	49	206	3.18×10^{-5}	1.98×10^{-3}	1.55×10^{-2}
Portion a_1 (−)	49	57.5	-7.28×10^{-6}	-0.94×10^{-3}	-1.082×10^{-2}
Areal					
Annulus					
1, $\alpha = 2\pi$	28.3	45	24.9×10^{-4}	2.44×10^{-1}	1.76
2, $\alpha = 4.38$	45	75.5	7.0×10^{-4}	1.14×10^{-1}	1.28
3, $\alpha = 3.78$	75.5	123.5	2.11×10^{-4}	0.65×10^{-1}	0.98
4, $\alpha = 3.44$	123.5	252	0.80×10^{-4}	0.61×10^{-1}	1.16
5, $\alpha = \pi$	252	∞	0.23×10^{-4}	0.60×10^{-1}	10.49
"Point"		216	4.7×10^{-10}	4.4×10^{-7}	1.1×10^{-6}

Assumptions: $h = \sqrt{20^2 + 20^2} = 28.3$ km; $\beta = 1.6$; $m_0 = 4$

Peak acceleration: $b_1 = 2000$; $b_2 = 0.8$; $b_3 = 2$; $C = 2.4 \times 10^9$

Peak velocity: $b_1 = 16$; $b_2 = 1.0$; $b_3 = 1.7$; $C = 4.98 \times 10^4$

Peak displacement: $b_1 = 7$; $b_2 = 1.2$; $b_3 = 1.6$; $C = 7.8 \times 10^3$

TABLE 1, PART 2

NUMERICAL EXAMPLE

Source	$\hat{\nu}_i G_i$		
	A	V	D
Line 1			
Right portion	5.12×10^{-11}	1.73×10^{-8}	3.66×10^{-7}
Left portion	10.6×10^{-11}	3.17×10^{-8}	5.99×10^{-7}
Line 2			
Total	318×10^{-11}	19.8×10^{-8}	15.5×10^{-7}
Portion a_1 (−)	-72.8×10^{-11}	-9.4×10^{-8}	-10.82×10^{-7}
Areal			
Annulus			
1, $\alpha = 2\pi$	249×10^{-11}	24.4×10^{-8}	17.6×10^{-7}
2, $\alpha = 4.38$	70×10^{-11}	11.4×10^{-8}	12.8×10^{-7}
3, $\alpha = 3.78$	21.1×10^{-11}	6.5×10^{-8}	9.8×10^{-7}
4, $\alpha = 3.44$	8.0×10^{-11}	6.1×10^{-8}	11.6×10^{-7}
5, $\alpha = \pi$	2.3×10^{-11}	6.0×10^{-8}	104.9×10^{-7}
"Point"	4.3×10^{-11}	4.1×10^{-8}	9.8×10^{-7}
Sum	616×10^{-11}	73.7×10^{-8}	182×10^{-7}

Numerical Example

For illustration we treat the hypothetical situation shown in Figure 8. The site is located on a deep alluvial plane (shaded) such that the geological structure below the site and to the south and east is not known in detail. Historically, earthquakes have occurred throughout this plane, but not often enough to determine fault patterns. The engineer chooses to treat the region as if the next earthquake were equally likely to occur in any unit area. The average rate, $\hat{\nu} = 1.0 \times 10^{-6}$ per km^2, was estimated by dividing the region's total number of earthquakes (with magnitudes in excess of 4) by its total area.

The exception, historically, is a small area, some 200 km southeast. It is also below the alluvial plane. The frequency of all sizes of earthquakes there has been relatively high, including several of larger magnitudes. Although the engineer can easily account for any suspected local difference in the parameter β (smaller values imply higher relative frequencies of larger magnitudes), he chooses to use the same β value, 1.6, for the entire region. In other words, he chooses to attribute the small area's observed larger magnitudes to the same population $f_M(m)$. The justification is that the larger the average arrival rate, the larger is the number of observations and the more likely it is that larger magnitudes will be included among the observations of a given period of time. Exactly what area (here shown as 30 by 30 km) is used to estimate the areal occurrence rate, $\hat{\nu} = 1.0 \times 10^{-4}$ per km^2, is not critical in this case since the area is small enough and far enough from the site that the entire source will be treated as a point with rate $\hat{\nu} = 0.09$.

Finally, to the northwest where the geological structure is exposed two faults have been located. Neither can be assumed inactive. Past activity on the first (and other geologically similar faults) suggests an average occurrence rate of $\hat{\nu} = 1.0 \times 10^{-4}$ per km. No earthquakes on the second, closer fault have been recorded, but its geological similarity to the first suggests that it be given a similar activity level.

The sectors of annuli used to represent the areal region are shown in Figure 8. The geometry factors, G_i, for the various sources are shown in Table 1 along with the products $\hat{\nu}_i G_i$, and their sums for peak-ground acceleration, A, peak-ground velocity V, and peak-ground displacement, D. The conclusion is that the maximum ground acceleration, velocity, and displacement during an interval of t years have distributions

$$F_{A_{\max}}(a) = \exp\left[-14.7ta^{-2}\right]$$

$$F_{V_{\max}}(v) = \exp\left[-0.0367tv^{-1.6}\right]$$

$$F_{D_{\max}}(d) = \exp\left[-0.142td^{-1.33}\right].$$

The annual maxima have the approximate distributions

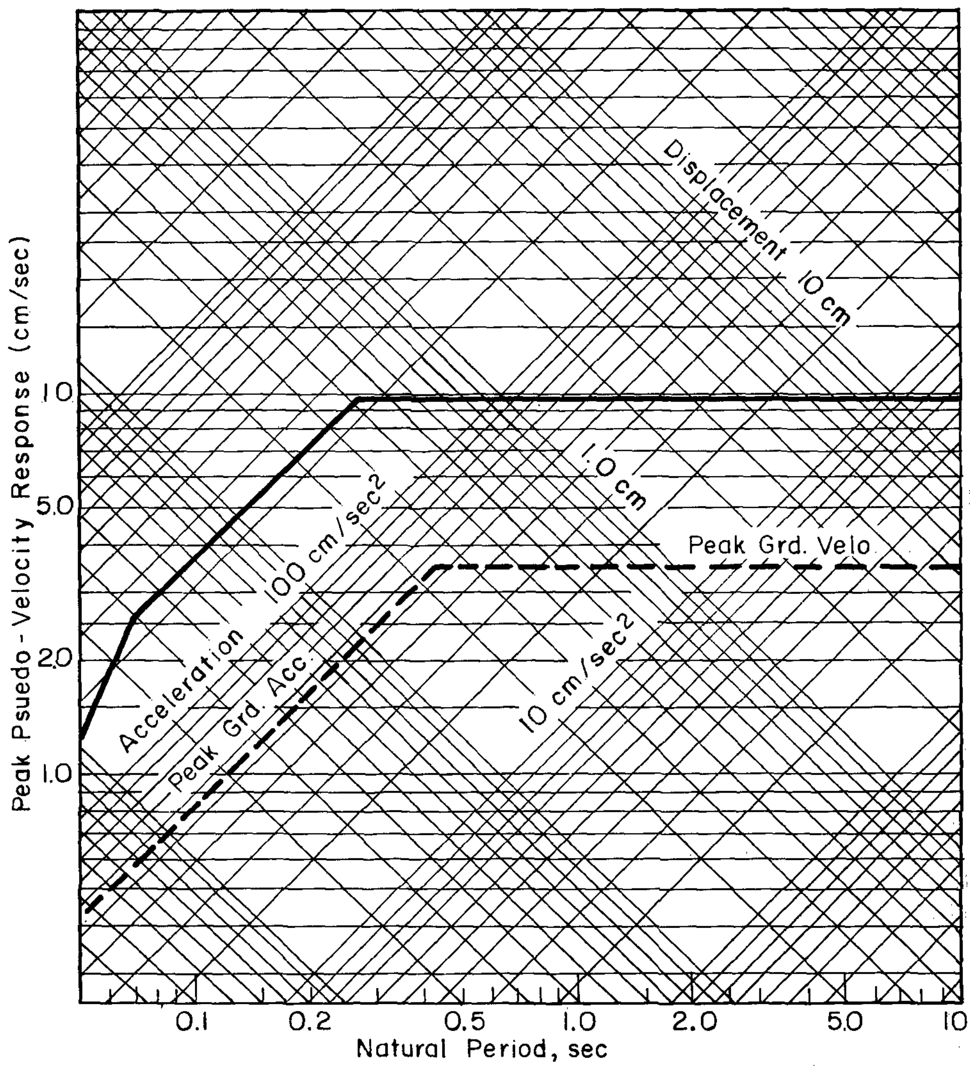
$$1 - F_{A_{\max}}(a) \cong 14.7a^{-2}$$

$$1 - F_{V_{\max}}(v) \cong 0.0367v^{-1.6}$$

$$1 - F_{D_{\max}}(d) \cong 0.142d^{-1.33}.$$

In terms of return periods $T_a \cong 0.0681a^2$, $T_v \cong 27.3v^{1.6}$, $T_d \cong 7.05d^{1.33}$, or $a \cong 3.83T_a^{\,0.5}$, $v \cong 0.126T_v^{\,0.625}$, $d \cong 0.231T_d^{\,0.75}$.

If a design response spectrum based on a 200-year return period were desired, it should be based on design ground-motion values of $a_{200} = 55$ cm/sec^2 $= 0.054g$, $v_{200} = 3.5$ cm/sec, $d_{200} = 12.5$ cm. Using the method for constructing a spectrum suggested for design by Newmark (1967), the dynamic response spectrum in Figure 9 is obtained.



Design Response Spectrum; 2 % of Critical Damping

FIG. 9. Dynamic response spectrum.

Notice that, for a proportional increase in all ground-motion factors, the risk (as measured by $1 - F$ or $1/T$) decreases more rapidly for peak acceleration than for peak velocity, and more rapidly for velocity than for displacement. The implication is that shorter-period structures can achieve greater risk reductions for the same percentage increase in design level than longer-period structures.

Inspection of the individual "contributions" to $\Sigma\nu_i G_i$ (or, approximately, to the risks $1 - F$) in Table 1 reveals that the closer faults have the predominant influence on peak acceleration risks, since even relatively small, more frequent magnitudes can give rise to high accelerations locally.* More distant potential sources contribute significantly to the risk of longer-period structures, as evidenced by their contributions to the $\Sigma\nu_i G_i$ factor for velocity and displacement. The slow decay with distance of peak displacement causes a large contribution even from long distances. This explains why the displacement of 12.5 cm is considerably larger than those associated with specific earthquakes records with peak accelerations of the order of 0.05g.

Assumptions and Extensions

While the assumptions made in the method are considered reasonable for most purposes of engineering design, a number of them can be relaxed without significantly altering the basic method. In particular, the distribution of magnitudes and the relationships used to relate site ground-motion characteristics to the magnitude and focal distance can be replaced with ease, only the results of certain integrations will change.

In the derivations above the distribution of magnitudes has been assumed to be the unlimited exponential distribution. For the larger, rarer magnitudes there are insufficient data to substantiate with confidence this or any other assumption (Rosenblueth, 1964). The shape as well as the parameters may in fact vary among different regions for these larger values. The magnitudes of earthquakes may be bounded. Relatively clean analytical results can be obtained for distribution functions of polynomial form and for the limited exponential distribution. Their influence, which may be significant for larger return periods, are under investigation.

For different focal distance relationships, the existing results can be used with piecewise fits to the other functions. For example, if it is assumed that there is no attenuation of Y with distance for a certain distance, r', from a source (Housner, 1965; Ipek *et al*, 1965) an annular source can be broken into two regions, one for $d \leqq r \leqq r'$ and the other for $r' \leqq r \leqq r_0$. In the first region b_3 should be set equal to zero, and the values of b_1 and b_2 appropriate for near-source conditions adopted. Coupled with a limited magnitude distribution, this process facilitates incorporation of any suspected upper bounds on maximum ground motions (Housner, 1965).

These functions are, in any case, no better than the parameter estimates used in them. One primary advantage of an analytical method, as opposed to a numerical one (Ipek *et al*, 1965; Lacer, 1965) is that the sensitivity of final conclusions to the accuracy of these parameter estimates can be assessed.

Other of the more basic assumptions in the method can also be relaxed with relative ease. Specifically, these include the two assumptions (a) that the radiation of effects can be treated as if the earthquake generating mechanism were concentrated at a point and (b) that isoseismals are circular. These assumptions are commonly made in design studies. This is done not so much because it is thought to be true, but because alternative methods and information are seldom available. The vast majority of statistical data on attenuation and scaling laws, for example, are available in forms (averages, etc.) based on these two assumptions. At the expense of added mathematical complexity alternative assumptions (e.g., finite mechanism length, elliptical isoseismals) can be incorporated into the method above if sufficient data are available to justify their inclusion.

*Also, of course, the durations are correspondingly short, a factor not explicitly appearing in the method for construction of response spectra proposed by Newmark.

The more fundamental assumptions are those of (a) equal likelihood of occurrence along a line or over an areal source, (b) constant-in-time average occurrence rate of earthquakes, and (c) Poisson (or "memory-less") behavior of occurrences.

If data or judgement rule against the equal-likelihood assumption and in favor of other relative values they can be included by simply treating each portion of the source over which the equally likely assumption is reasonable as an individual source using the superposition method described above.

If the engineer and the seismologist are prepared to make an assumption about the time dependence of the average occurrence rate, other than that of constant in time, a minor modification in the method suffices to account for this non-homogeneity in time (Parzen, 1962; Cornell, 1964). The influence appears, for example, in Equation 21 as

$$P\,[I_{\max}^{(t)} \leqq i] = \exp\left[-\, p_i \int_0^t \nu(\tau)\; d\tau\right] \tag{59}$$

in which $\nu(\tau)$ is the average occurrence rate at time τ.

The assumption that the occurrences of earthquakes follow the behavior of the Poisson process model can be removed only at a greater penalty, however. The Poisson assumption does not reflect earthquake swarms or aftershocks, nor is it physically consistent with the elastic rebound theory, which implies that a zone of recent past activity is less likely to be the source of the next earthquake than a previously active zone which has been relatively quiet for some time. These limitations can, in principle, be removed by adopting more general renewal process or Markov process models (Aki, 1956; Vere-Jones, 1966). For engineering purposes the Poisson results are considered adequate for numerous reasons (Rosenblueth, 1966; Lomnitz, 1966). When swarms and aftershocks are excluded, data does not clearly reject the Poisson assumption (Lomnitz, 1966; Wanner, 1937; Knopoff, 1964; Niazi, 1964) for the rarer, major events of engineering interest. Even when more accurate theoretical models become available, it is not evident that sufficient statistical data and other information will be available in many regions to permit the seismologist to adopt a non-Poisson assumption or to estimate any more parameters than the average occurrence rates.

The structural engineer is concerned more directly with a design response spectrum. For random forcing functions such as earthquake ground motions, the duration of motion also influences the peak-response values (Rosenblueth, 1964; Crandall and Mark, 1963). Given a relationship between duration and M and R, and given a function relating (expected) peak response to duration and to expected peak-ground acceleration or velocity, a simple application of the same method will produce such response spectra. In addition, inclusion of the randomness of peak response to random motions with given parameters (Rosenblueth, 1964; Crandall and Mark, 1963) will permit the construction of response spectra based on prescribed probabilities of responses not to be exceeded in a given lifetime. There is strong reason to believe that this latter influence is negligible (Rosenblueth, 1964; Borges, 1956).

Although developed specifically for the seismic risk analysis of individual sites, the method systematically applied to a grid of points would yield regional seismic probability maps. These might take a form similar to those used in determining design winds (Thom, 1967), namely contours of maximum ground motion of equal return period. Consistent maps could be produced to as fine a scale as desired. Perhaps the greatest advantage of this method for this purpose is that it would insure that consistent assumptions were being used for all portions of the region and among different regions.

All assumptions made by the seismologists involved would be explicit and quantitative, open to review and to up-dating with new evidence. Major difficulties would remain, however, in the judgement of active sources, in the estimation of their average activity rates, and in determination of local soil influence.

Conclusion

A quantitative method of evaluating the seismic risk at a particular site has the advantage that consistent estimates of these risks can be prepared for various potential sites, all perhaps in the same general region but in significantly different geometrical relationships with respect to potential sources of earthquakes.

Such a method is necessary to determine how rapidly the risk decays as the resistance of the system's design is increased. Reasonable economic trade-offs, be they with respect to operating regulations, below-standard performance, or system malfunction, cannot be made without such quantitative relationships.

The method proposed offers the means by which to make these engineering analyses consistent with the seismicity information available. This information is transferred from the seismologist in the form of his best estimates of the average rate of seismic activity of potential sources of earthquakes, the relative likelihoods of various magnitudes of events on those sources, and the relationships between site characteristics, distance, and magnitude applicable for the region.

The conclusions appear in an easily applied, easily interpreted form, suitable for review for consistency and sensitivity to assumptions.

For the most commonly assumed functional forms of the relationships used, the upper tails of the probability distributions of the design ground motion parameters are found theoretically to be of Type I or Type II extreme value type.

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