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ESTIMATION OF THE EARTHQUAKE RECURRENCE PARAMETERS FOR UNEQUAL OBSERVATION PERIODS FOR DIFFERENT MAGNITUDES

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ABSTRACT

Maximum likelihood estimation of the earthquake parameters N_o and β in the relation $N = N_o \exp(-\beta m)$ is extended to the case of events grouped in magnitude with each group observed over individual time periods. Asymptotic forms of the equation for β reduce to the estimators given for different special cases by Aki (1965), Utsu (1965, 1966), and Page (1968). The estimates of β are only approximately chi-square distributed. For sufficiently large numbers of events, they can be estimated from the curvature of the log-likelihood function.

Sample calculations for three earthquake source zones in western Canada indicate that for well-constrained data sets, the often-used, least-squares estimation procedures lead to compatible results, but for less well-defined data sets, the effect of subjective plotting and weighting methods used for least-squares fitting leads to appreciably different parameters.

INTRODUCTION

Recent requirements of seismic risk estimation have led to a re-evaluation of historical earthquake records and statistical methods in many countries, with a view to optimizing the use of the available information. Whatever approach is chosen to quantify risk, the basic information is earthquake catalogs from which a recurrence relation is derived. Its most widely used form is still the Gutenberg-Richter log-linear relation, $\log N = a - bm$, perhaps with some modification at larger magnitudes.

The estimation of the parameters, especially b, has received much attention. The basic premise for the use of the conventional least-squares method is violated in this case, especially if N is the cumulative event count. The least-squares method dates back to Gauss (cf. e.g., Kendall and Stuart, 1963, p. 71), who derived it intuitively, but also recognized that it was the maximum likelihood method for data that are independent and whose error distributions follow the "Gaussian", or normal error law. However, cumulative event counts are not independent, and the number of earthquake occurrences are better represented by a Poisson rather than a Gaussian distribution. Furthermore, weighted least squaring does not invalidate these basic objections to the method, and in fact, relies upon additional unjustifiable assumptions.

The maximum likelihood estimation of b was discussed by Aki (1965) who gave a formula equivalent to

$$\frac{1}{\beta} = \bar{m} - m_o, \tag{1}$$

where $\beta = b \ln(10)$, $\bar{m}$ is the average magnitude of the sample, and m_o is the lowest magnitude at which event observations are complete. Utsu (1965) derived the same estimator for β by equating the first moments of the population and the sample.

Equation (1) applies to continuous magnitude values. However, event magnitudes can rarely be specified more accurately than to a $\frac{1}{4}$ magnitude unit, often only to $\frac{1}{2}$ unit and it is, therefore, common practice to group events into classes with equal magnitude increments. For such grouping, with half-width δ, the estimate of β from equation (1) is biased and Utsu (1966) tabulates a correction factor which modifies (1)

$$\frac{1}{\beta}\frac{\beta\delta}{\tanh(\beta\delta)} = \overline{m} - m_o. \tag{2}$$

A realistic risk analysis must admit a regional maximum possible magnitude, even though it may not yet be possible to estimate this magnitude reliably. Lacking compelling evidence for more complicated forms, a simple truncation of the Gutenberg-Richter recurrence relation is suggested, preferably of the incremental form, since a truncation of the cumulative relation implies a spike in the recurrence density. Page (1968) considered this modification and gives a maximum likelihood estimate for β, for data with continuous magnitudes between m_o and m_x, as

$$\frac{1}{\beta} = \overline{m} - m_o - \frac{m_x \exp(-\beta(m_x - m_o))}{1 - \exp(-\beta(m_x - m_o))}. \tag{3}$$

Error estimates for β were given by both Aki and Utsu. Aki (1965) uses the central limit theorem to arrive at a Gaussian distribution of β around its maximum likelihood estimate, β_o, with a standard deviation of $\beta_o N^{-1/2}$. This should not be used for small N. Also, Aki tabulates values for $N \geqq 50$. Utsu (1966) gives $1/\beta$ as chi-square distributed, with $\chi^2 = 2N\beta_o/\beta$ and the number of degrees of freedom $f = 2N$.

Current applications of seismic risk for critical engineering structures, i.e., nuclear reactors, make it desirable to optimize the use of available data in every justifiable way. For instance, the seismic risk estimates included in the current (1977) Canadian National Building Code are derived from formal calculations based on a 76-yr data period (Milne and Davenport, 1969), even though information for the largest magnitude earthquakes in eastern Canada is considered complete over about 300 yr, while $m4$ earthquakes may only be cataloged completely since the 1920's in that region (cf. Basham *et al.*, 1979). Stepp (1972) has also discussed the utilization of unequal observational periods for different magnitudes and tests for completeness at each magnitude. Molchan *et al.* (1970) recognize the same problems, but use n_i/T_i, event numbers divided by time interval of completeness for each magnitude interval, as maximum likelihood estimator. These authors do not impose a maximum magnitude. More details on the Russian work can be found in Kantorovich *et al.* (1970).

Generalization to Unequal Observational Periods

Ignoring the possibly very serious question of time variability of earthquake activity, the following generalization and combinations of earlier work appear desirable: (a) unequal observational periods, t_i; (b) grouping of data in magnitude classes, $m_i \pm \delta$; and (c) an imposed maximum magnitude, m_x.

The periods of observation are independently determined, e.g., by Stepp's (1972) method or from a consideration of historical seismograph capability (Basham *et al.*, 1979; Milne *et al.*, 1978). Similarly, the regional maximum earthquake must be independently estimated from geophysical considerations, such as maximum fault lengths, regional stress drop, and earthquake history.

With the arbitrary choice of a truncated recurrence density, the probability of an earthquake having its magnitude between m and $m + dm$ is

$$\begin{aligned} p(m)\, dm &= \text{const.}\beta\, e^{-\beta m}\, dm \quad m_o \leqq m \leqq m_x \\ &= 0 \quad\quad\quad\quad \text{otherwise.} \end{aligned} \tag{4}$$

Integration over magnitude intervals and proper normalization leads to the likelihood function, L, for n_i events in magnitude class m_i

$$L(\beta \mid n_i, m_i, t_i) = \frac{N!}{\prod_i n_i!} \prod_i p_i^{n_i} \tag{5}$$

where $p_i = \dfrac{t_i \exp(-\beta m_i)}{\sum_j t_j \exp(-\beta m_j)}$. An extremum of $\ln(L)$ is obtained for

$$\frac{\sum_i t_i m_i \exp(-\beta m_i)}{\sum_j t_j \exp(-\beta m_j)} = \frac{\sum n_i m_i}{N} = \bar{m} \tag{6}$$

which can easily be solved for β by an iterative scheme (e.g., Newton's method). A computer program is given in the Appendix.

It is interesting to compare the asymptotic forms of equation (6) with the corresponding earlier equations. For equal observational periods, $t_i = t$, $m_1 = m_o + \delta$ and number of intervals $\dfrac{m_x - m_o}{2\delta}$, this reduces to

$$\frac{1}{\beta}\left[\frac{\beta\delta}{\tanh(\beta\delta)} - \frac{\beta\dfrac{m_x - m_o}{2}}{\tanh\left(\beta\dfrac{m_x - m_o}{2}\right)}\right] = \bar{m} - \frac{m_x + m_o}{2}. \tag{7}$$

For large m_x, this reduces to equation (2), Utsu's formula for grouped data without an upper magnitude bound. As δ goes to 0, all n_i become unity, and Page's result is obtained, which, in our notation, is

$$\frac{1}{\beta} = \bar{m} - \frac{m_x + m_o}{2} + \frac{(m_x - m_o)/2}{\tanh(\beta(m_x - m_o)/2)}. \tag{8}$$

Equations (2), (7), and (8), require recursive solutions which make them no more useful than the general equation (6).

A simple, almost intuitive estimate of the variance of β can be obtained from the curvature of $\ln(L)$ at its maximum. The greater the curvature, the sharper the maximum and the smaller the variance. For instance, for a set of Gaussian observations with likelihood $L = \Pi \exp\left[\dfrac{-(x_i - \bar{x})^2}{2\sigma^2}\right]$, and with an estimate for $\bar{x}$ given by $\dfrac{\partial \ln L}{\partial \bar{x}} = \sum_i \dfrac{(x_i - \bar{x})}{\sigma^2} = 0$, which is the unweighted least-squares estimate, one finds $\dfrac{-\partial^2 \ln L}{\partial \bar{x}^2} = \dfrac{N}{\sigma^2}$. This is the usual expression for the reciprocal of the variance of the mean, except that a factor $(N-1)/N$ is included if a sample estimate, s^2, is used instead of the unknown σ^2. For the likelihood function leading to equation (1), N/β^2 is identical to Aki's result. More generally, the law of large numbers ascertains (e.g., Edwards, 1972) that, for sufficiently large numbers, β is approximately normally distributed about its mean with variance

$$\operatorname{var}(\beta) = -\left(\frac{\partial^2 \ln L}{\partial \beta^2}\right)^{-1}$$

Another derivation and result are given by Kendall and Stuart (1963). Equation (6) yields

$$\operatorname{var}(\beta) = \frac{1}{N} \frac{\left[\sum_i t_i \exp(-\beta m_i)\right]^2}{\left[\sum_i t_i m_i \exp(-\beta m_i)\right]^2 - \sum_i t_i \exp(-\beta m_i) \sum_i t_i m_i^2 \exp(-\beta m_i)} \tag{9}$$

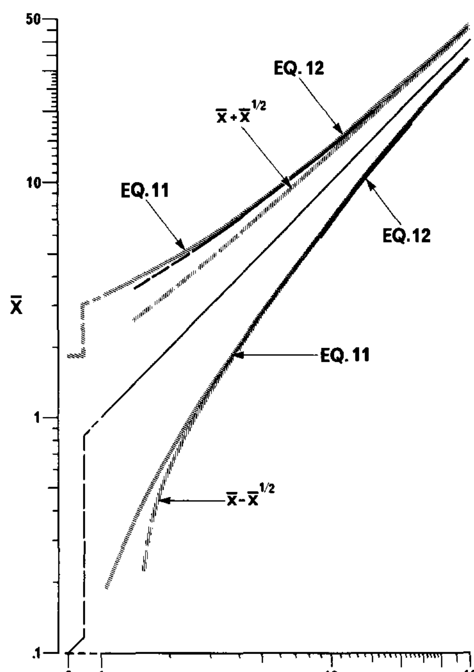

FIG. 1. ± 1 S.D. confidence intervals, 15.9 and 84.1 percentiles, for an estimated average $\bar{x}$ of a Poisson variable, calculated from equations (11) and (12) and from $\bar{x} + \sqrt{x}$.

which is obtained as a by-product from solving (6) numerically using the Newton iteration scheme (cf. Appendix).

For a useful estimate of β, the total number of events, N, should be large enough to allow use of a Gaussian with variance (9) for the distribution of β, but Utsu's chi-square distribution with $\chi^2 = 2N\beta_o/\beta$, $f = 2N$ could be used for small numbers. Utsu includes this cumulative distribution of b_o/b as his Figure 1. From this figure and the results shown in our Figure 1, the approximation may well be used with sufficient accuracy to much smaller N. However, it should be pointed out that this distribution, in fact, disagrees with the maximum likelihood principle, since its peak does *not* occur at $\chi^2 = 2N$ [cf. also the discussion following equation (11)].

The activity parameter, equivalent to "a" in the Gutenberg-Richter relation, is not usually discussed since its maximum likelihood estimate is simply the total number of events observed above the threshold of completeness. For unequal observation periods, the total number of events, N, is still a Poisson variable, being the sum of Poisson variables, each given by the sought-after actual annual event rate, N_a, at or above m_o, multiplied by the probability, q_i, of falling into magnitude increment m_i, $q_i = \exp(-\beta m_i)/\sum_j \exp(-\beta m_j)$, and multiplied by the number of years which each magnitude increment was observed [q_i is not the same as p_i in equation (5)]. Thus,

$$N_a = N \sum_i \exp(-\beta m_i)/\sum_j t_j \exp(-\beta m_j). \tag{10}$$

For identical t_i, this reduces appropriately to N/t.

The variance of N_a is N_a/N, and since N hopefully is a substantial number, confidence limits can approximately be obtained from the normal distribution. However, the chi-square distributions quoted in the next paragraph are more appropriate for smaller N.

For visual display and comparison, earthquake recurrence data are usually plotted,

TABLE 1

CONFIDENCE INTERVALS FOR POISSON MEAN, N*

μ_U	N	μ_L
1.84	0	0
3.30	1	0.173
4.64	2	0.708
5.92	3	1.37
7.16	4	2.09
8.38	5	2.84
9.58	6	3.62
10.8	7	4.42
12.0	8	5.23
13.1	9	6.06
14.3	10	6.89

* Lower and upper ±1 S.D. confidence intervals, i.e., 15.9 and 84.1 percentiles use μ_L and μ_U from equation (11). Above $N = 9$, use Figure 1 or $N - N^{1/2}$ for the lower bound and $N + 3/4 + (N + 1/2)^{1/2}$ from equation (12).

most often as logarithmic cumulative event counts, accumulated from above. Regardless of the method of calculation, lines are then shown which should fit the data in a convincing manner. The obvious way of quantifying the expected fit is the inclusion of error bars, e.g., ±1 S.D. above or below the plotted points. For large event numbers, $\pm N^{1/2}$ corresponds closely to the usual interpretation as a confidence interval of ±1 S.D., enclosing 68 per cent of the distribution. For small numbers and for annual rates, especially when derived from unequal observation intervals, questions may arise about the equivalent confidence intervals to use, and the following expressions are, therefore, quoted here for the lower and upper limits, μ_L and μ_U of the two-sided intervals of confidence 1-α in terms of chi-square distributions with f degrees of freedom (Graf *et al.*, 1966), and their numerical values for ±1 S.D. are given in Table 1

$$\mu_L = \tfrac{1}{2}\,\chi^2_{\alpha/2;f} \qquad \text{with } f = 2N$$

$$\mu_U = \tfrac{1}{2}\,\chi^2_{1-(\alpha/2);f} \quad \text{with } f = 2(N+1). \tag{11}$$

This is obviously not an exact result. For instance, only the distribution for the upper limit has its maximum at N, in agreement with the maximum likelihood approach used to estimate N in the first place. Furthermore, for small confidence, i.e., for the practically not useful case of $\alpha/2$ approaching 0.5, the change in f leads to a finite confidence range. On the other hand, for null observations, equation (11) gives useful nontrivial upper limits. For $N \gtrsim 9$, approximations for (11) are given by

$$\begin{aligned} \mu_L &= N - \tfrac{1}{2} + \tfrac{1}{4}\, u^2_{1-\alpha/2} - u_{1-\alpha/2}\sqrt{(N-1/2)} \\ \mu_U &= N + \tfrac{1}{2} + \tfrac{1}{4}\, u^2_{1-\alpha/2} + u_{1-\alpha/2}\sqrt{(N+1/2)} \end{aligned} \tag{12}$$

where u_α is the Gaussian variate for confidence α.

Figure 1 shows the limits derived from equations (11) and (12) for 68.2 per cent confidence as well as $N \pm N^{1/2}$. Over the whole range, down to $x = 4$ or 3, the lower limit (11) is better approximated by $N - N^{1/2}$ than by (12) which is only shown for its recommended range. For the upper limit (11), one finds (12) to be the better approximation, but again $N + N^{1/2}$ is useful, although not conservative, down to about $x = 10$.

Confidence bounds for earthquake *rates*, from equal or unequal observation periods, are obtained for the relevant total event count and scaled down to unit time.

TABLE 2

NUMBER OF EVENTS IN 1/4 MAGNITUDE INCREMENTS, AND THE CORRESPONDING OBSERVATION PERIODS BEFORE AND INCLUDING 1975 FOR SOME OF THE WESTERN CANADIAN ZONES OF EARTHQUAKE OCCURRENCE

	Magnitudes, Interval Centers														
Zone	4.0		4.5		5.0		5.5		6.0		6.5		7.0		7.5
No. of events															
Cascades	3	6	1	2	2	0	0	0	1						
Puget Sound	9	6	1	1	5	0	1	2	0	0	1	0	1		
North Vancouver Island	1	1	1	0	0	0	1	1	1	0	2	0	1	1	
No. of years	←		25		→	←	45	→	←				76		→

A SAMPLE APPLICATION

The described estimation procedure is currently used to determine seismicity parameters for the Canadian zones of earthquake occurrence, such as used by Basham *et al.* for eastern Canada, where magnitude 6 to 7 may be completely cataloged over 300 yr while magnitude 3.5 has only recently become complete. As an illustration of the method and its remaining problems, an application to some of the western Canadian zones is presented here. Table 2 lists the condensed data that were used. These were abstracted from the Canadian earthquake catalog, and grouped in $\frac{1}{4}$ magnitude intervals. A grouping error is incurred which will be discussed later.

A test of the Poisson assumption was not made; in fact, it is expected to be this assumption is violated, because it is difficult to define and remove aftershock sequences. On the contrary, with a view toward obtaining conservative activity estimates, it could be argued that aftershocks should be counted, in case of doubt. This, as well as ignoring a possible time variability of earthquake activity, will result in overly optimistic error estimates.

Figure 2 shows incremental and cumulative rates with ±1 S.D. error bounds. The

84 percentiles are also shown for the empty magnitude intervals: they depend only on the length of the respective observation periods. The least-squares lines are minimizations for log(N or n) residuals. Other methods such as minimization of perpendicular distances from the lines, or a nonparametric method (cf. Weichert and Milne, 1979) give slightly different lines, but this is inconsequential for this comparison.

The maximum likelihood estimate is calculated from numbers of earthquakes in magnitude intervals, with the assumptions of a log-linear recurrence relation and

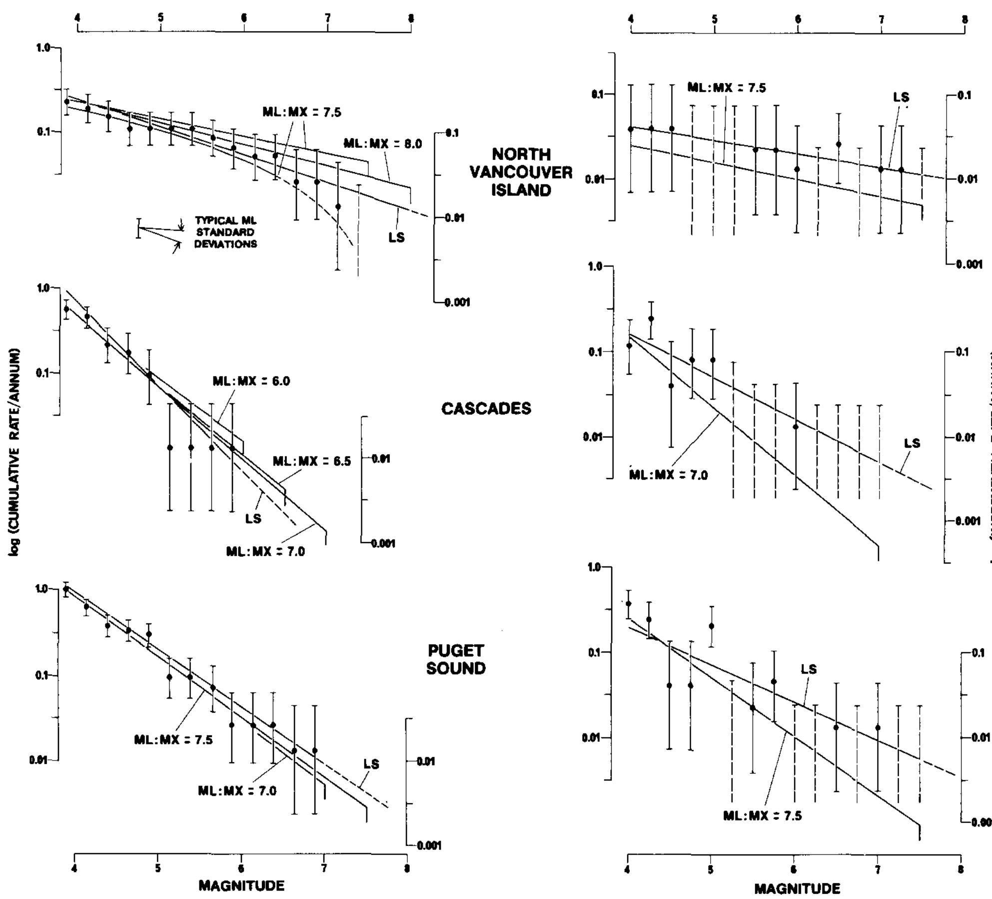

FIG. 2. *Right*: incremental plots of earthquake rates, with least-squares lines (LS) and the maximum likelihood (ML) equivalent lines for the assumed regional maximum magnitude (MX) earthquake. *Left*: cumulative plots of earthquake rates, with least-squares lines and maximum likelihood lines for several maximum magnitudes. Typical ±1 S.D. estimates for the ML parameters are indicated. The properly curved cumulative ML estimate is only shown for one extreme example.

straight cutoff of the event density at some maximum magnitude; therefore, a visual comparison with the data and with least-squares estimates should be made on an incremental plot, even though here, the information that was contained in the different lengths of the observation periods has already disappeared except, perhaps, for the unexpectedly broad confidence intervals at lower magnitudes. For each of the seismic source zones, the maximum likelihood line is shown for one maximum magnitude in addition to the least-squares lines.

One notes that the least-squares recurrence slopes are all shallower than the corresponding maximum likelihood slopes. This is due to the points at minus infinity (log 0) representing the empty intervals and predominantly occurring at the high-magnitude end. These points are not taken into account by the least-squares method. In the case of the North Vancouver Island Zone, empty magnitude intervals are well distributed over most of the magnitude range, pulling the ML estimate below all plotted data points. In general, both the least-squares and maximum likelihood lines fit the incremental plot equally well, according to the simple criterion of expecting the line to pass through about $\frac{2}{3}$ of the 68 per cent confidence intervals.

For the least-squares method, the unpleasantness of empty intervals and large data scatter is partly overcome by plotting and fitting on a cumulative plot. The customary repetition of points from right to left results in a rather arbitrary high weighting of the less well-established rates in the higher magnitude range, but also has an unintentional beneficial effect described below.

In comparing cumulative and incremental recurrence curves, one must remember two points. First, the level of the lines will depend on both β and the interval width. For the usual plotting convention of placing cumulative points at the lower end of the respective magnitude interval and centering incremental points, one finds, well away from M_x, where the density is truncated, that the incremental/cumulative ratio, n/N, is approximately given by

$$n/N = \exp(\beta\delta) - \exp(-\beta\delta) = 2\sinh(\beta\delta). \tag{13}$$

The straight lines denoted by ML:MX = $\cdots$, in the cumulative plots, are related to the maximum likelihood estimates for the respective M_x by this expression.

As the maximum magnitude is decreased and empty intervals above the observed data range are omitted, the maximum likelihood lines become shallower, often by an appreciable fraction of their standard deviations. However, for the Cascade and Puget Sound seismic zones, no clear bias between least-squares and maximum likelihood lines can be recognized. One can conclude that least-squares fitting in cumulative plots is a reasonable approximation for well-defined data sets. It appears that the repetition of points for empty intervals tends to pull the estimates down, while in the incremental form this is not possible. Other weighting schemes could be used, such as $1/N$, but conceptually none can compete with maximum likelihood. It is noted that the formulation given here does *not* give an estimate of M_x, but alternative approaches could be used such as McGuire (1977).

The second, more important point to observe is the curvature that should be shown in the cumulative equivalent of the incremental maximum likelihood lines as a result of the cutoff at M_x. Thus, the cumulative curves, N', should go to minus infinity according to the relation

$$\log(N') = \log(N\exp(-\beta M)(1 - \exp(-(M_x - M)))). \tag{14}$$

This effect is small enough for representative β of 1.6 ($b = 0.7$), to be ignored in Figure 2 for the Cascades and Puget Sound seismic zon(•, for the purposes of comparison with least-squares straight lines. However, for the low β of the North Vancouver Island Zone, the effect is so pronounced, that the ML:MX = 7.5 line lies above all data points. The properly calculated curve is, therefore, shown for this case. As expected from the incremental plot, it passes *below* the data.

In formulating the statistical approach, it was assumed that catalog magnitudes could be grouped unambiguously. This is not true, since the older earthquake magnitudes were given to $\frac{1}{4}$ magnitudes, so that the increment used for this illustration is appropriate. For $m0.1$ catalog increments, $m\frac{1}{4}$ grouping leads to unequal magnitude intervals; in this case, a $m0.5$ grouping would be more appropriate as, e.g., used by Basham *et al.* and by Milne *et al.* (1978). However, this makes the assignment of the older $m\frac{1}{4}$ earthquakes ambiguous. Further generalizations of the estimation procedure to take this into account do not appear worthwhile in the light of the available data.

Conclusions

It is suggested that estimation of recurrence parameters in the Gutenberg-Richter relation should always employ a maximum likelihood method, and the method presented here gives the necessary extension of known results to the important case of unequal periods of observation. Although for well-constrained data, the use of alternative methods, such as least squares may lead to equivalent results, one finds that for less well-defined data sets, the effect of subjective plotting and weighting methods leads to appreciably different parameters. In particular, least-squares fitting does not allow inclusion of a preconceived judgment on maximum magnitude, and also ignores the information content of the empty magnitude intervals, even though, in a cumulative plot, an empirical partial correction is usually made. Finally, it should perhaps be pointed out, that uncertainty of the lower cutoff magnitude affects results of all methods in a similar way, but this problem is not addressed in this paper.

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Appendix

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C       ESTIMATION OF BETA BY MAXIMUM LIKELIHOOD FOR VARIABLE OBSERVATION
C       PERIODS FOR DIFFERENT MAGNITUDE INCREMENTS.
C       OTHER VALUES PRINTED: B AND ST. DEV., N5 AND ST. DEV. AND LOG(NO)
C
C       INDEX OF LOWEST AND HIGHEST MAGNITUDE GROUP TO BE USED
C       IS K=LOW,IGH
C       IT(K) = LENGTH OF OBSERVATION PERIOD OF MAGNITUDE K.
C       FMAG(K)=CENTRAL VALUE OF MAGNITUDE INCREMENT K.
C       N(K)= NUMBER OF EVENTS IN MAGNITUDE INCREMENT K.
C       DATA INPUT AT DISCRETION OF USER
C
        DIMENSION IT(21),FMAG(21),N(21),TITLE(20)
        BETA = 1.5        ! INITIAL TRIAL VALUE
C       ITERATION LOOP:
77      CONTINUE
        SNM=0.
        NKOUNT=0
        STMEX=0.
        SUMTEX=0.
        STM2X=0.
        SUMEXP=0.
        DO 2 K=LOW,IGH
        SNM = SNM+N(K)*FMAG(K)
        NKOUNT=NKOUNT+N(K)
        TJEXP=IT(K)*EXP(-BETA*FMAG(K))
        TMEXP=TJEXP*FMAG(K)
        SUMEXP=SUMEXP+EXP(-BETA*FMAG(K))
        STMEX=STMEX+TMEXP
        SUMTEX=SUMTEX+TJEXP
        STM2X=STM2X+FMAG(K)*TMEXP
```

```
2       CONTINUE
        DLDB=STMEX/SUMTEX  ! *N - SUMNM = 0  FOR EXTREMUM
        D2LDB2 = NKOUNT*(DLDB**2 - STM2X/SUMTEX)
        DLDB = DLDB*NKOUNT-SNM
        BETL=BETA
        BETA = BETA - DLDB/D2LDB2
        STDV = SQRT(-1./D2LDB2)
        B=BETA/ALOG(10.)
        STDB=STDV/ALOG(10.)
        FNGTMO=NKOUNT*SUMEXP/SUMTEX
        FN5=FNGTMO*EXP(-BETA*(5.-(FMAG(LOW)-0.125)))
        FN0=FNGTMO*EXP( BETA*(   FMAG(LOW)-0.125 ))
        FLGN0=ALOG10(FN0)   !  FOR 0.25 INCREMENTS
        STDFN5=FN5/SQRT(FLOAT(NKOUNT))
        IF(ABS(BETA-BETL).GE..0001) GO TO 77
        PRINT 200,BETA,STDV,B,STDB
200     FORMAT(13X,' BETA=',F8.4,' +/-1 STDV OF',F7.3,'  B='
        1,F8.4,' +/-1 STDV OF',F7.3/)
        PRINT 210,NKOUNT,FLGN0,FN5,STDFN5
210     FORMAT(14X,'TOTAL NUMBER OF EVENTS ',I4,
        1', LOG(ANNUAL RATE ABOVE M0) ',F6.3/
        2 14X,'ANNUAL RATE ABOVE M5 ',F8.4,' +/-1 STDV OF'F7.3)
        STOP
        END
```

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